Enrollment No: ________________________ **Exam Seat No:**________________________

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name : Engineering Mathematics-I

Subject Code : 4TE01EMT2 **Branch :** B.Tech (All)

Semester : 1 **Date :** 02/12/2015 **Time :** 10:30 To 1:30 **Marks :** 70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: (1 marks each) **(14)**

a) n^{th} derivative of $y=\frac{1}{x+a}$ is

(a) $\frac{(-1)^n n!}{(x+a)^{n+1}}$ (b) $\frac{(-1)^{n-1} n!}{(x+a)^{n+1}}$ (c) $\frac{(-1)^n n!}{(x+a)^{n}}$ (d) none of these

b) If $y=\sin^{-1} x$ then x equal to

(a) $1-\frac{y^2}{2!}+\frac{y^4}{4!}-......$ (b) $y-\frac{y^3}{3!}+\frac{y^5}{5!}-......$ (c) $1+y+\frac{y^2}{2!}+\frac{y^3}{3!}+......$

(d) none of these

c) A square matrix A is called Singular if

(a) $|A|=0$ (b) $A^2=A$ (c) $AA^T=I$ (d) $|A|\neq 0$

d) What is the value of y_3? where $y=\sin 2x$

(a) $8\sin 2x$ (b) $-8\sin 2x$ (c) $-8\cos 2x$ (d) $8\cos 2x$

e) For $n\times n$ non homogeneous system of equations $AX=B$,
If $\rho(A)=\rho(A:B)<n$ then the system has

(a) No solutions (b) Unique solutions
(c) Infinite solution (d) None of these

f) Rank of the matrix $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 6 & 8 \end{bmatrix}$ is ______

(a) 0 (b) 1 (c) 2 (d) 3

g) State De-Moivre's theorem.

h) Separate $sinh(x + iy)$ into real and imaginary parts

i) $e^{i\pi/2} =$ ______

$(a)\ 0 \quad (b)\ 1 \quad (c)\ i \quad (d)\ -1$

j) State Euler's theorem for homogeneous function.

k) Find $\frac{dy}{dx}$ for $x^2 + y^2 - xy = 0$

l) $\frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(u,v)} =$ _____

m) State L-Hospital's rule to evaluate Indeterminate forms.

n) If $y = e^{ax}$ then y_n ______

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions

A Reduce the matrix $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$ to the normal form and find rank. **(05)**

B State the Euler's theorem on homogeneous function and use it to prove that **(05)**

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \tan^3 u, \text{ where } u = \sin^{-1}\left(\sqrt{x^2 + y^2}\right).$$

C Evaluate $\lim_{x \to 0} \left(\frac{a^x + b^x}{2} \right)^{\frac{1}{x}}$. **(04)**

Q-3 Attempt all questions

A Test for consistency and if possible solve the equations **(05)**

$3x + 3y + 2z = 1,\ x + 2y = 4,\ 10y + 3z = -2,\ 2x - 3y - z = 5$

B If $y = a\cos(\log x) + b\sin(\log x)$ then prove that **(05)**

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0.$$

C If $z = x^2 \tan^{-1}\left(\frac{y}{x}\right)$, find $\frac{\partial^2 z}{\partial x \partial y}$. **(04)**

Q-4 Attempt all questions

A Find the Eigen values and the corresponding Eigen vectors for the matrix **(05)**

$$\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$$

B Prove that $(1 + i\sqrt{3})^8 + (1 - i\sqrt{3})^8 = -(2)^8$. **(05)**

C If $x = e^v \operatorname{cosec} u$, $y = e^v \cot u$, find $\frac{\partial(x, y)}{\partial(u, v)}$. **(04)**

Q-5 Attempt all questions

A Find the inverse of the following matrix $\begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ by Gauss-Jordan method. **(05)**

B If α and β are roots of equation $z^2 - 2\sqrt{3}z + 4 = 0$ then prove that **(05)**

$\alpha^n + \beta^n = 2^{n+1} \cos\frac{n\pi}{6}$.

C Find the n^{th} derivative of $y = \log\left(x + \sqrt{1+x^2}\right)$. **(04)**

Q-6 Attempt all questions

A Examine for linear dependence or independence of vectors (2,3,4,-2) , (-1,-2,-2,1) and (1,1,2,-1). Hence find the relation between them, if dependent. **(05)**

B Solve $x^7 + x^4 + i\,(x^3 + 1) = 0$ using De-Moiver's theorem. **(05)**

C Expand $tan^{-1}x$ in powers of $\left(x - \frac{\pi}{4}\right)$. **(04)**

Q-7 Attempt all questions

A Prove that $\sinh^{-1} x = \log\left(x + \sqrt{x^2+1}\right)$. **(05)**

B Express $sin^8\theta$ in a series of cosines of multiples of θ. **(05)**

C Find $\frac{dy}{dx}$, if $\sin(xy) = e^{xy} + x^2 y$. **(04)**

Q-8 Attempt all questions

A Examine for extreme values for the function $x^2 + y^2 + 6x + 12$. **(05)**

B Separate into real and imaginary parts $\sqrt{i}^{\sqrt{i}}$. **(05)**

C Using Taylor's series, arrange $x^3 - 3x^2 + 4x + 3$ in power of $(x - 2)$. **(04)**